A Robust Mixed-Integer Convex Model for Optimal Scheduling of Integrated ES-SOP Devices

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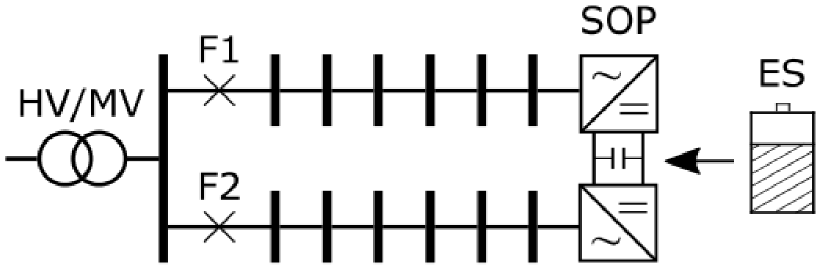
From Newcastle. For the world.

Summary: a method is developed to schedule integrated energy storage-SOP devices that is robust to uncertainty without compromising on model complexity

Drivers of storage-SOP devices in networks:

- Technology push - developments in BESS and power converters
- Market pull - distribution network capacity needed in next decade



Questions:

- How should we model the full ES-SOP system to given multiple services?
- How might a DSO schedule such a system in a constrained distribution network?

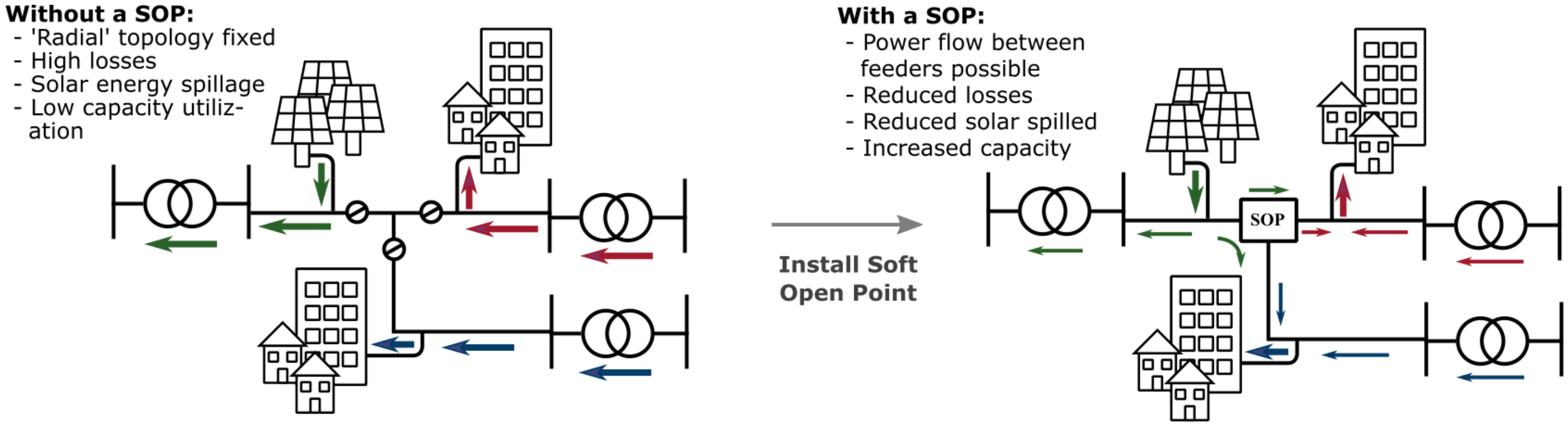
This paper:

A tractable, robust method for scheduling ES-SOP devices subject to renewable and load forecast uncertainties. Three contributions:

- Robust optimization formulation for the ES-SOP system,
- using a binary-polynomial su,bsystem loss model,
- as a tractable, mixed-integer convex program

$$D^t\left(\bar{\mathbf{d}}^t, \hat{\mathbf{d}}^t, \Gamma^t\right) := \left\{ \begin{aligned} & \mathbf{d}^t \in \mathbb{R}^{|\Omega_n|} : \sum_{i \in \Omega_n} \left(\theta_{i,t}^+ + \theta_{i,t}^-\right) \le \Gamma^t, \theta_{i,t}^+ + \theta_{i,t}^- \le 1, \\ & d_i^t = \bar{d}_i^t + \hat{d}_i^t \theta_{i,t}^+ - \hat{d}_i^t \theta_{i,t}^-, \ \forall i \in \Omega_n \end{aligned} \right\}$$

Soft Open Points (SOPs) use AC-DC-AC converters to enable power transfer between feeders in distribution grids, providing a variety of potential grid services



This work: **energy storage is connected to the DC link of the SOP**, a win-win:
- Energy storage can make use of SOP converters;
- The SOP can provide both spatial and temporal demand shifting

Source: author's own

ES-SOP system modelling: a “binary-polynomial model” for modelling efficiency is introduced to better match subsystem physics and operating characteristics

Benchmark subsystem loss model:

$$P_{\text{Loss}} = k|S|$$

Proposed subsystem loss model:

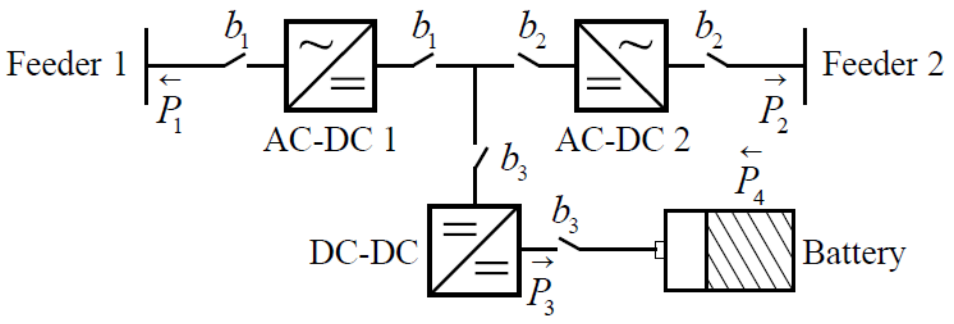
$$P_{\text{Loss}} = b(c_0 + c_1|S| + c_2|S|^2)$$

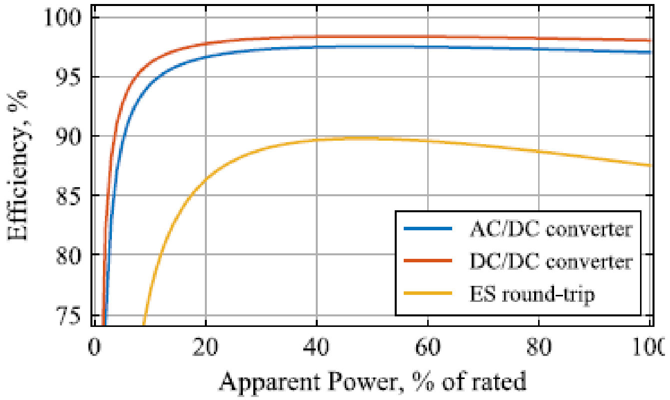
where

- $b \in \{0,1\}$ is the on-off state of the subsystem
- c_0 represents turn-on losses (e.g., filters, transformers)
- c_1 diode-like losses
- c_2 ohmic losses

Notes:

- Different components have different forms
  - E.g., Battery has $b = 1$ and $c_1 = 0$
- Model can be fit to measured converter data
- Efficiency typically peaks at partial load
- Can be represented in mixed-integer conic form





The proposed adaptive optimization robust optimization approach could allow DSOs to dispatch assets with varying degrees of robustness and recourse

The proposed two-stage approach is based on the formulation of Bertsimas et al:

- First stage:
  - Initial decisions made (e.g., committing units / scheduling ES)
- Second stage
  - 'Outcome' ($\boldsymbol{d}$)-aware feasibility and objective (e.g., reserve dispatch given $\boldsymbol{d}$; impact of ES-SOP schedule $\boldsymbol{x}$ on network losses)
- 'Budget of uncertainty' Γ controls size of uncertainty set D

This work:

- First stage $\boldsymbol{x}$ includes ES-SOP scheduling
- Second stage $\boldsymbol{y}$ is only network variables (including losses)
- Budget of Uncertainty Γ chosen based on Monte Carlo heuristic to minimize Probability of Constraint Violation (PoCV)

$$
\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{y}(\cdot)} \quad & \left(\boldsymbol{c}^T \boldsymbol{x} + \max_{\boldsymbol{d}\in\mathcal{D}} \boldsymbol{b}^T \boldsymbol{y}(\boldsymbol{d}) \right) \\
\text{s.t.} \quad & \boldsymbol{F}\boldsymbol{x} \leq \boldsymbol{f},\ \boldsymbol{x} \text{ is binary} \\
& \boldsymbol{H}\boldsymbol{y}(\boldsymbol{d}) \leq \boldsymbol{h}(\boldsymbol{d}),\ \forall \boldsymbol{d} \in \mathcal{D} \\
& \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y}(\boldsymbol{d}) \leq \boldsymbol{g},\ \forall \boldsymbol{d} \in \mathcal{D} \\
& \boldsymbol{I}_u \boldsymbol{y}(\boldsymbol{d}) = \boldsymbol{d},\ \forall \boldsymbol{d} \in \mathcal{D}
\end{aligned}
$$

Bertsimas et al 2013

Source: Bertsimas et al, "Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem", IEEE Transactions on Power Systems, 2013

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & \left(\mathbf{c}^{\mathrm{T}}\mathbf{x} + \max_{\mathbf{d}\in D} \min_{\mathbf{y}\in\Omega(\mathbf{x},\mathbf{d})} \mathbf{e}^{\mathrm{T}}\mathbf{y} \right) \\
\text{s.t.} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b},\ \left\| \mathbf{F}\mathbf{x} \right\|_2 \leq \mathbf{f}^{\mathrm{T}}\mathbf{x}
\end{aligned}
$$

This work

where

$$
\Omega(\mathbf{x},\mathbf{d}) = \left\{ \mathbf{y} : \mathbf{H}\mathbf{x} + \mathbf{K}\mathbf{y} = \mathbf{d},\ \mathbf{M}\mathbf{y} \leq \mathbf{r},\ \left\| \mathbf{G}\mathbf{y} \right\|_2 \leq \mathbf{g}^{\mathrm{T}}\mathbf{y} \right\}
$$

$$
D^t\left(\bar{\mathbf{d}}^t, \hat{\mathbf{d}}^t, \Gamma^t\right) := \left\{ \begin{array}{l} \mathbf{d}^t \in \mathbb{R}^{|\Omega_n|} : \sum_{i\in\Omega_n} \left(\theta_{i,t}^{+} + \theta_{i,t}^{-}\right) \leq \Gamma^t, \theta_{i,t}^{+} + \theta_{i,t}^{-} \leq 1, \\ d_i^t = \bar{d}_i^t + \hat{d}_i^t \theta_{i,t}^{+} - \hat{d}_i^t \theta_{i,t}^{-},\ \forall i \in \Omega_n \end{array} \right\}
$$

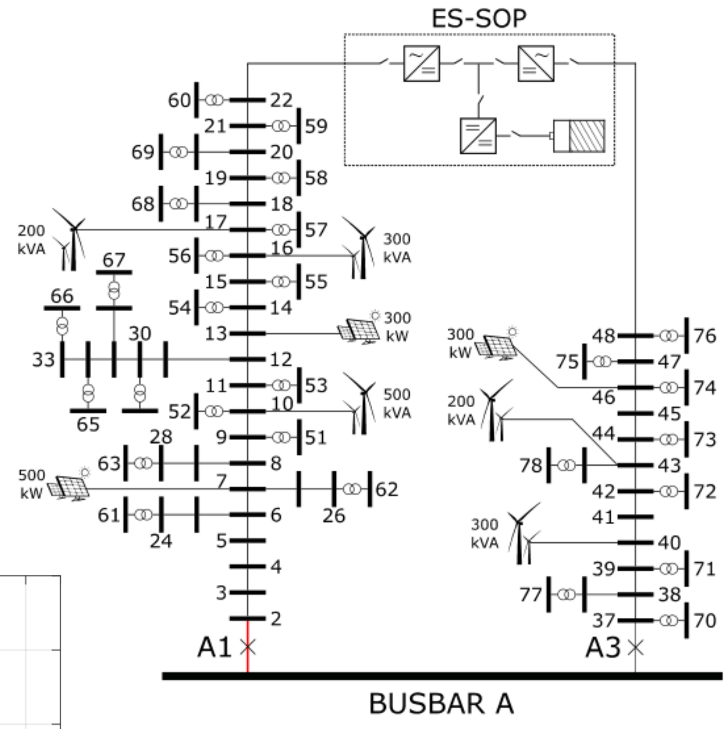
A test case and a real-world network are considered to test the proposed ES-SOP device in, using combined loss reduction-arbitrage as the cost function

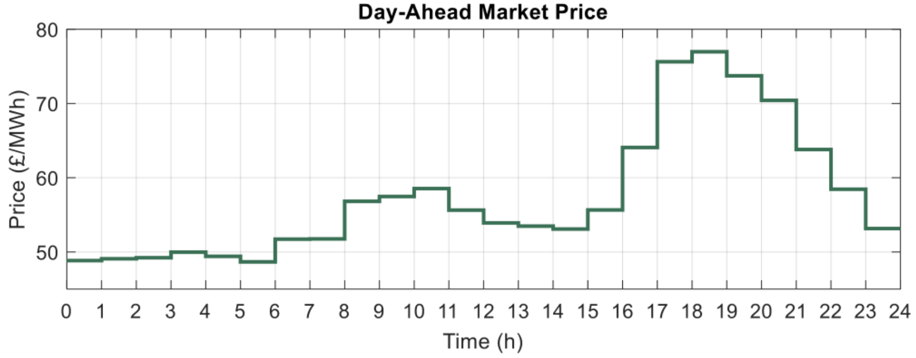
- Network power flow conic relxation (Farivar and Low)

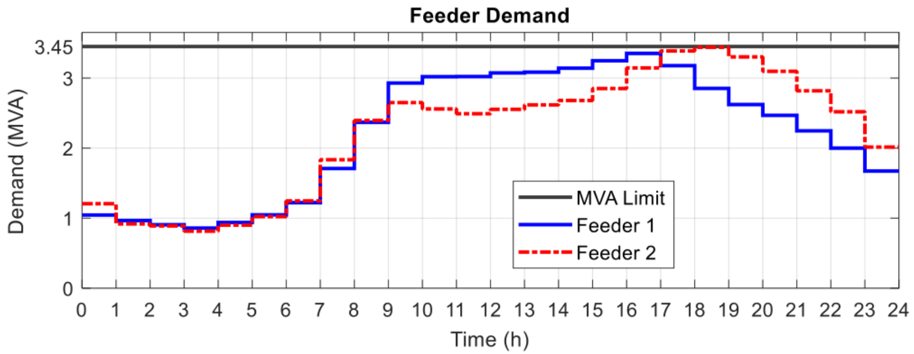
- Two case studies:
  - IEEE 33 Bus (connected end-to-end) – a lossy network with poor power factor and congestion
  - North East distribution system – lower level of congestion; and lower losses

- +/- 10% uncertainty for each load and generator, uniform distribution

- 400 kVA SOP; 400 kW/800 kWh BESS







Results 1 of 3: it is possible to pick out qualitative differences between the solution of the proposed and modelled approach

- At low power the marginal losses of the ac/dc converters are lower, allowing an additional ES cycle
- High ES export during the evening peak is suppressed
- Improvements allow improvements in both arbitrage, network loss and ES-SOP losses
  - NB no-ES SOP loss cost of £162.7

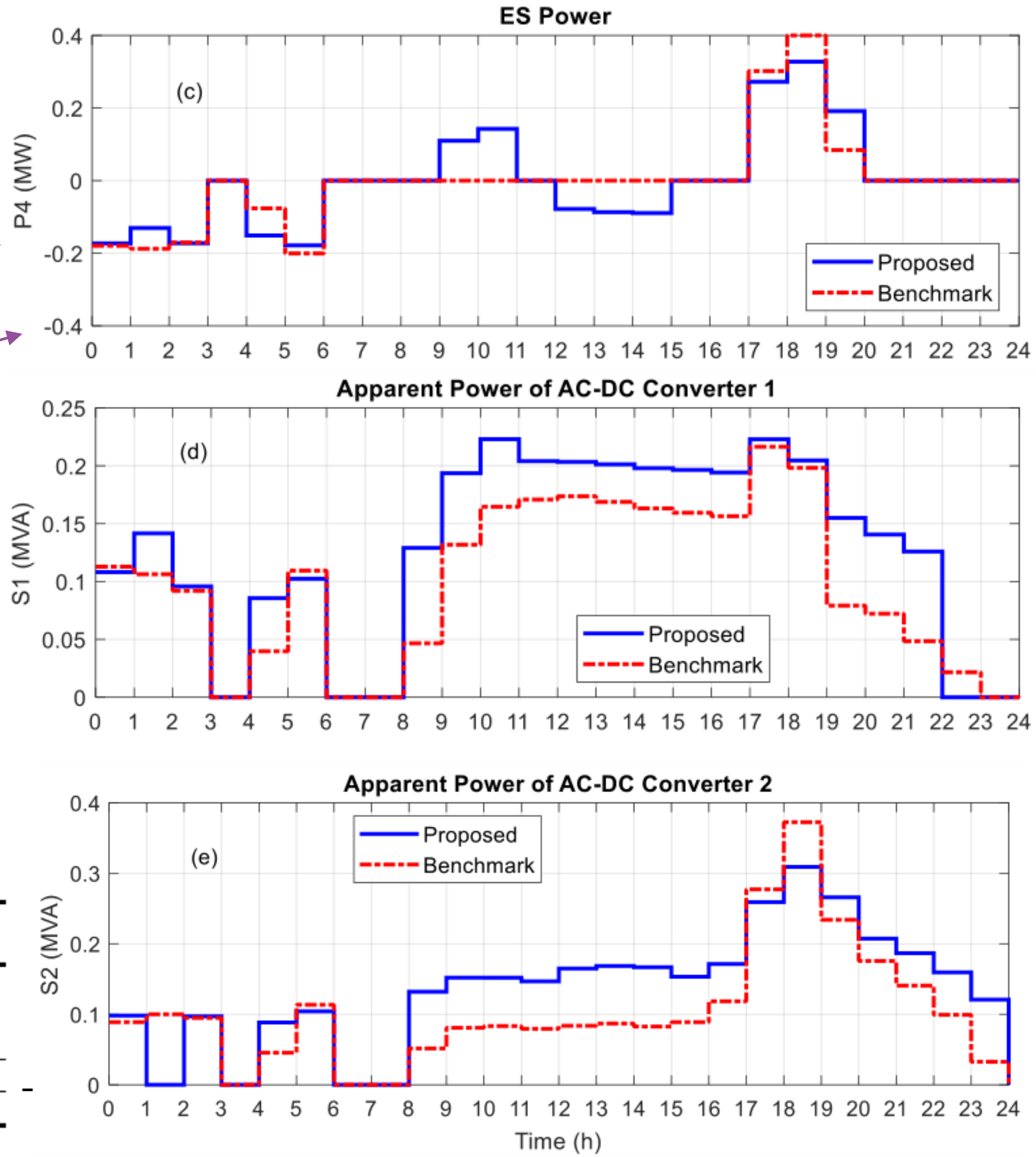
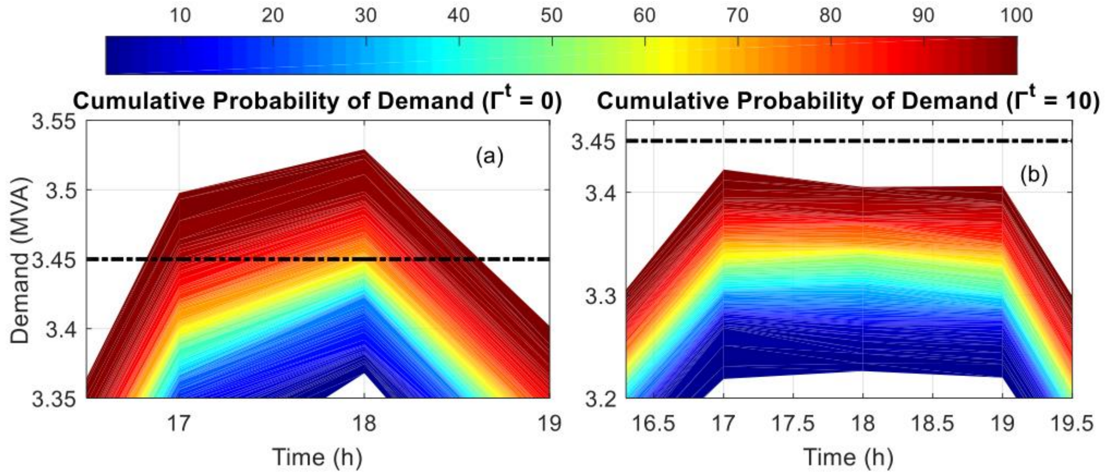
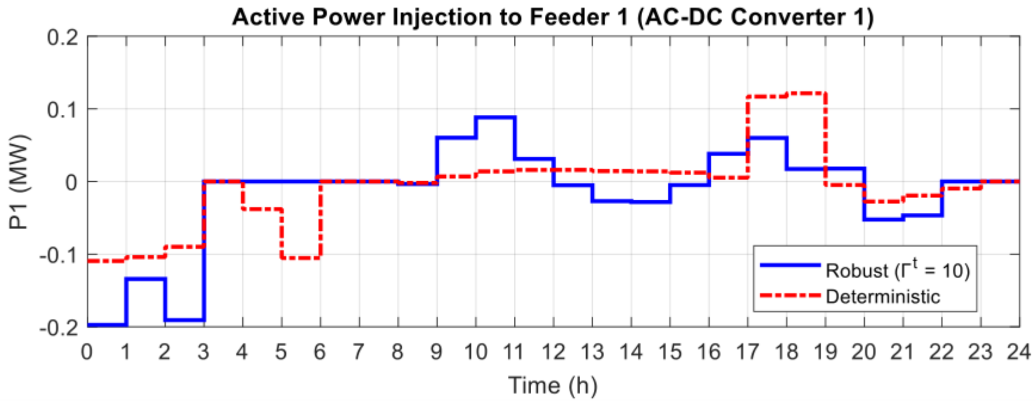


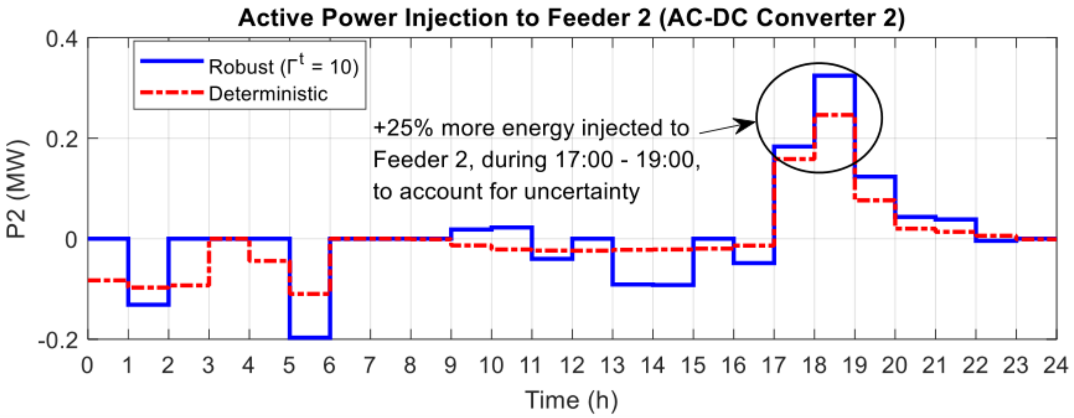
TABLE III
COMPARISON OF OUR PROPOSED MODEL WITH BENCHMARK [7]

	Proposed Model	Benchmark [7]	Difference (%)
Network Loss Cost (£)	145.8	148.1	-
ES-SOP Loss Cost (£)	14.6	12.9 (**14.9**)	-
Arbitrage Profit (£)	-21.5	-19.9	-
Total (£)	138.9	141.1 (**143.1**)	-
Saving (£)	23.8	21.6 (**19.6**)	+21.4%

Results 2 of 3: by tweaking the budget of uncertainty, a more robust solution is found







Results 3 of 3: the method performs well as compared to alternative approaches, and shows good performance on average

- Difference between proposed and benchmark ES-SOP greater in real world network (lower network losses)

TABLE VIII
COMPARISON OF OUR PROPOSED MODEL WITH BENCHMARK [7] – REAL-WORLD DISTRIBUTION NETWORK

	Proposed Model	Benchmark [7]	Difference (%)
Network Loss Cost (£)	69.37	69.38	-
ES-SOP Loss Cost (£)	8.06	8.49 (**14.14**)	-
Arbitrage Profit (£)	-25.28	-24.75	-
Total (£)	52.15	53.12 (**58.77**)	-
Saving (£)	14.53	13.56 (**7.91**)	+83.7%

- Much more computationally efficient that stochastic optimization

TABLE VI
RESULTS OF TWO-STAGE STOCHASTIC PROGRAMMING I (OPTIMALITY GAP = 0.5%)

Number of Scenarios	Computational Time	Optimality Gap
2	1h	>2%
5	1h	>2.5%
10	No feasible solution found in 2h	
Robust	68s	0.5%

- Average price to provide robustness is (as compared to optimization value) is not much higher (implies difference largely due to network losses)

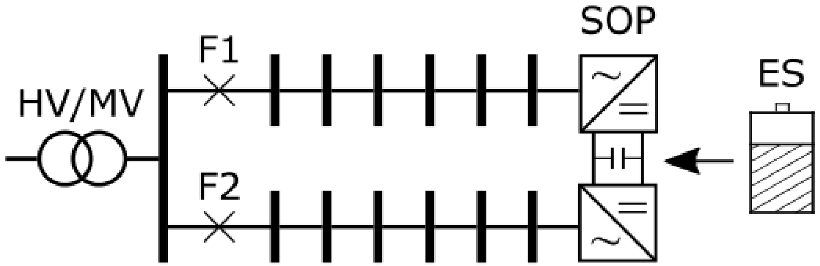
TABLE IX
POCV, ROBUST OPTIMAL VALUE, AND MEAN OBJECTIVE FUNCTION VALUE FOR DIFFERENT VALUES OF Γ – REAL-WORLD DISTRIBUTION NETWORK

Γ^t	PoCV	Rob. Opt. Value	Increase	Mean Obj. Fun. Value	Increase
0	8%	£52.15	0%	-	-
1	3.6%	£55.33	6.1%	£52.22	0.13%
2	1.5%	£56.98	9.3%	£52.40	0.47%
3	0%	£59.73	14.5%	£52.69	1.03%

Optimal Hybrid Multiplexed AC-DC-AC Converters

Conclusions

- Accurate converter loss modelling is necessary when considering total losses
- A two-stage adaptive robust optimization formulation can be used to schedule the ES-SOP device to avoid possible congestion
- Results show good performance can be achieved in an appropriate time (<< 30 mins)



Future work:

- Variations on this approach - other uncertainty models, recourse actions, etc to ensure match to DSO priorities and situation
- Modelling and analysis of reconfigurable ac/dc converters

$$D^t\left(\bar{\mathbf{d}}^t, \hat{\mathbf{d}}^t, \Gamma^t\right) := \left\{ \begin{array}{l} \mathbf{d}^t \in \mathbb{R}^{|\Omega_n|} : \sum_{i \in \Omega_n} \left(\theta^+_{i,t} + \theta^-_{i,t}\right) \le \Gamma^t, \theta^+_{i,t} + \theta^-_{i,t} \le 1, \\ d_i^t = \bar{d}_i^t + \hat{d}_i^t \theta^+_{i,t} - \hat{d}_i^t \theta^-_{i,t}, \ \forall i \in \Omega_n \end{array} \right\}$$

Thank you for your time!

Question, comments and thoughts always welcome:

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Start of research project

End of research project

MC2 PROJECT

Supergen Energy Networks

CESI National Centre for Energy Systems Integration

Solution method: the two-stage optimization is solved using Constraint-and-Column Generation (CCG)

Algorithm 1: Column and Constraint Generation

1. Set LB = $-\infty$, UB = $+\infty$, $m = 0$, tolerance ε.
2. **while** (UB - LB < ε) **do**
3. Solve the MP (61)-(67). Get optimal solution and objective, $\mathbf{x}^*$ and f_{m}, respectively. LB $\leftarrow$ max{LB, f_{m} }.
4. Given $\mathbf{x}^*$, solve the dual SP (76)-(83). Get worst-case uncertainty realization $\mathbf{d}^*$ and objective f_{s}. UB $\leftarrow$ min{UB, $\mathbf{c}^{\mathrm{T}}\mathbf{x}^* + f_{\mathrm{s}}$ }.
5. $\mathbf{d}^{m+1} \leftarrow \mathbf{d}^*$. Introduce new variables $\mathbf{y}^{m+1}$. Add constraints (64)-(67) to the MP.
6. $m \leftarrow m + 1$.
7. **end**
8. Return $\mathbf{x}^*$.

Master problem

$$f_{\mathrm{m}} = \min \ \mathbf{c}^{\mathrm{T}}\mathbf{x} + \eta$$
$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \le \mathbf{b}$$
$$\left\|\mathbf{F}\mathbf{x}\right\|_2 \le \mathbf{f}^{\mathrm{T}}\mathbf{x}$$
$$\eta \ge \mathbf{e}^{\mathrm{T}}\mathbf{y}^{v}, \ \forall v = 1, \ldots, m$$
$$\mathbf{H}\mathbf{x} + \mathbf{K}\mathbf{y}^{v} = \mathbf{d}^{v}, \ \forall v = 1, \ldots, m$$
$$\mathbf{M}\mathbf{y}^{v} \le \mathbf{r}, \ \forall v = 1, \ldots, m$$
$$\left\|\mathbf{G}\mathbf{y}^{v}\right\|_2 \le \mathbf{g}^{\mathrm{T}}\mathbf{y}^{v}, \ \forall v = 1, \ldots$$

Dual subproblem

$$f_s = \max_{\sigma^+, \sigma^-, \theta^+, \theta^-, \lambda, \pi, \omega, \varphi} \ \bar{\mathbf{d}}\boldsymbol{\lambda} + \hat{\mathbf{d}}\boldsymbol{\sigma}^+ - \hat{\mathbf{d}}\boldsymbol{\sigma}^- - \mathbf{H}\mathbf{x}^*\boldsymbol{\lambda} + \mathbf{r}\boldsymbol{\pi}$$
$$s.t. \quad \mathbf{K}\boldsymbol{\lambda} + \mathbf{M}\boldsymbol{\pi} + \sum_{h=1}^{H}\left(\mathbf{G}\boldsymbol{\omega} + \mathbf{g}\boldsymbol{\varphi}\right) = \mathbf{e}$$
$$\left\|\boldsymbol{\omega}\right\|_2 \le \boldsymbol{\varphi}$$
$$-M^+_{ij,t}\theta^+_{ij,t} \le \sigma^+_{ij,t} \le M^+_{ij,t}\theta^+_{ij,t}, \ \forall ij \in \Omega_{\mathrm{b}}, \ \forall t \in \Omega_{\mathrm{t}}$$
$$-M^+_{ij,t}\left(1-\theta^+_{ij,t}\right) + \lambda_{ij,t} \le \sigma^+_{ij,t} \le \lambda_{ij,t} + M^+_{ij,t}\left(1-\theta^+_{ij,t}\right)$$
$$-M^-_{ij,t}\theta^-_{ij,t} \le \sigma^-_{ij,t} \le M^-_{ij,t}\theta^-_{ij,t}, \ \forall ij \in \Omega_{\mathrm{b}}, \ \forall t \in \Omega_{\mathrm{t}}$$
$$-M^-_{ij,t}\left(1-\theta^-_{ij,t}\right) + \lambda_{ij,t} \le \sigma^-_{ij,t} \le \lambda_{ij,t} + M^-_{ij,t}\left(1-\theta^-_{ij,t}\right)$$
$$\boldsymbol{\lambda} \text{ free}, \ \boldsymbol{\pi} \le 0, \ \boldsymbol{\omega} \text{ free}, \ \boldsymbol{\varphi} \ge 0, \ \boldsymbol{\sigma}^+, \boldsymbol{\sigma}^- \text{ free}, \ \boldsymbol{\theta}^+, \boldsymbol{\theta}^- \in \{0,1\}$$