

# A Novel Formulation of LV Distribution Network Equivalents for Reliability Analysis

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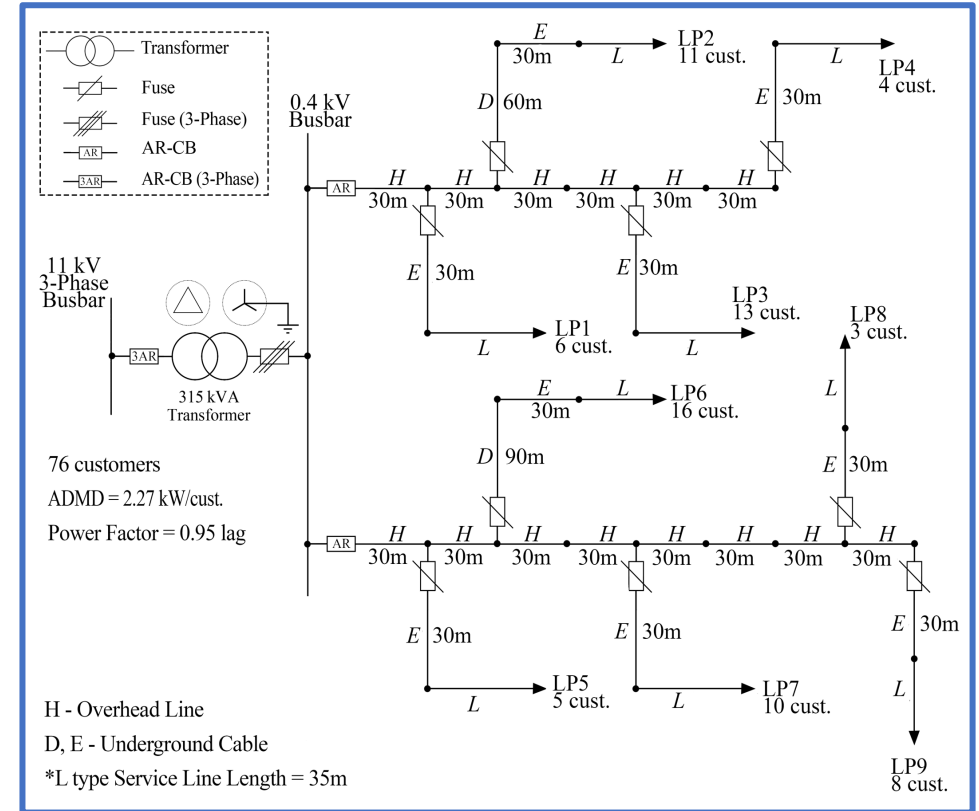
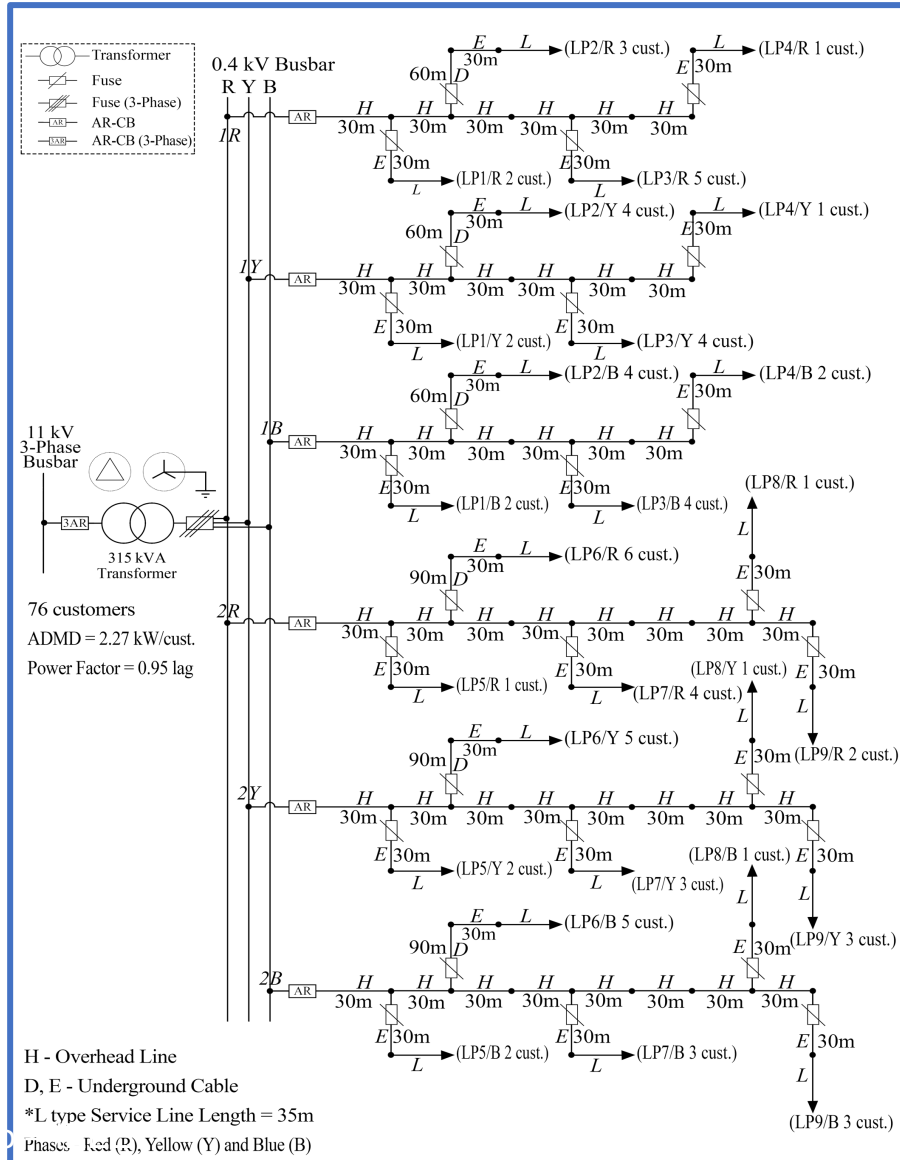
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# Single-Line (1-ph) vs Three-Phase (3-ph) Models

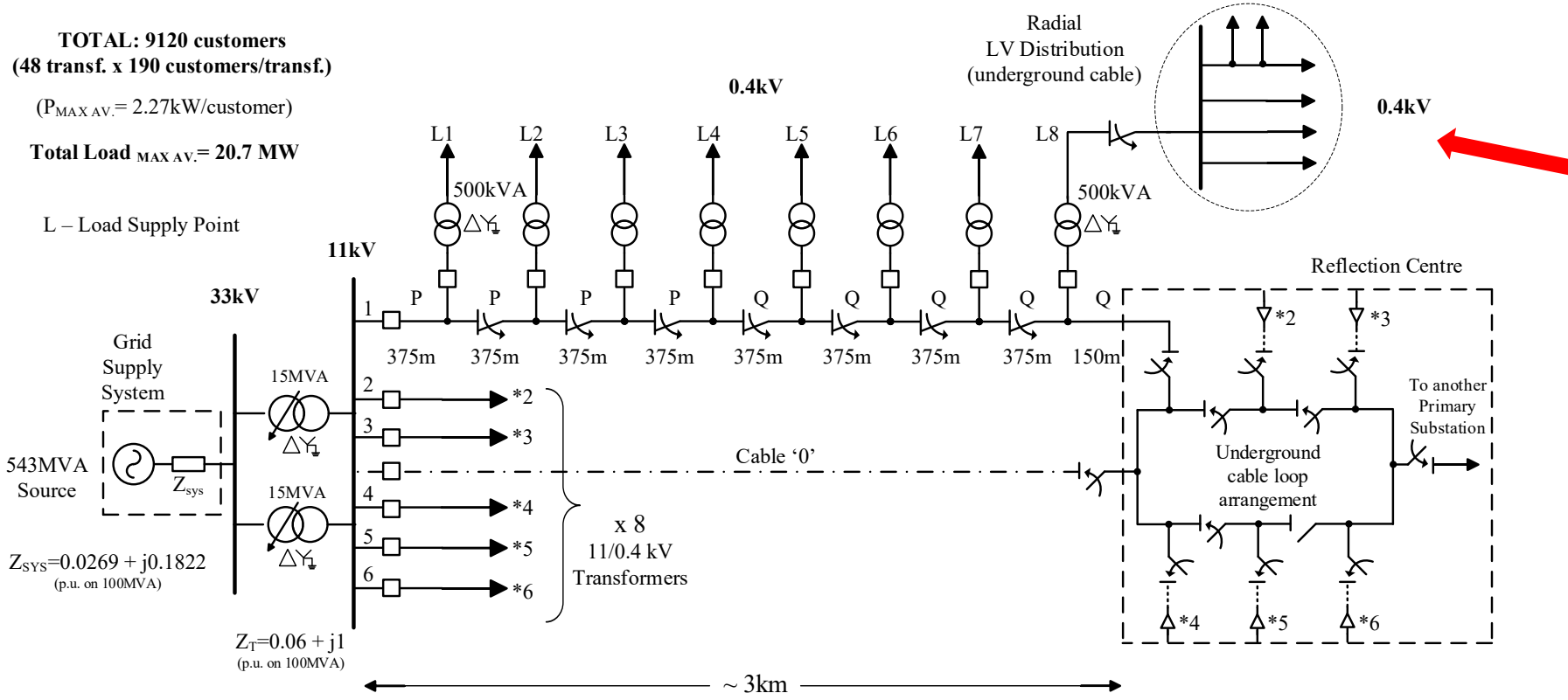
Three-phase model of an 'unbalanced' (& realistic) generic suburban LV network



Single-line model of a 'balanced' generic suburban LV network

Ignores phase-connection of customers, and protection system components (single-pole vs three-pole)

# Reliability Performance of MV networks



TOTAL: 9120 customers  
 (48 transf. x 190 customers/transf.)  
 $(P_{MAX AV.} = 2.27kW/customer)$   
 Total Load  $_{MAX AV.} = 20.7 MW$   
 L – Load Supply Point

Volume and complexity!

Use lumped aggregate model for each LV network with total load & number of customers

Reduce computation times for reliability assessment

Inaccurate representations

- Neglect the highly dispersed loads and
- different component types

- Large cumulative errors
- Incorrect characterisation of the quality of supply for different customers
- Affects network planning and operation

# State Enumeration

- **State Enumeration (SE)** is used to perform reliability analyses of large networks.
- Usually, for a system of  $m$  **repairable** components, there are  $2^m$  system states.
- Each state is a combination of the status of different components (UP/DOWN).
- It is **not computationally feasible** to enumerate all system states for large systems.
- Usually, the **analysis stops** at a given enumeration depth (ED), which corresponds to a failure level.

$$ED_k = C_0^m + C_1^m + \dots + C_k^m$$

For a system of  $k = 97$  components:

System states with:

$$ED_1 = C_0^m + C_1^m$$

$$ED1 = C_0^{97} + C_1^{97} = 98 \text{ states}$$

Only 1 component failure

$$ED_2 = C_0^m + C_1^m + C_2^m$$

$$ED2 = C_0^{97} + C_1^{97} + C_2^{97} = 4754 \text{ states}$$

Only 2 component failures

# State Enumeration cont'd

- The **frequency of occurrence** of each system state and the **mean duration** of residing in each state is based on the **failure and repair rates of components**.
- The impact of each selected state must be identified in terms of the interrupted loads/customers.
- Sequential **Monte Carlo Simulation (MCS)** can then be used to calculate reliability indices based on an artificial cycle of system operating and failure states.

## Considering only low-order contingencies

- Reduced computational efforts without significantly impacting accuracy
- Successive transitions between two system failure states are very rare.

## Considering only high-order contingencies

- System resilience
- Protection design & control

# Sequential Monte Carlo Simulation

- Sequential MCSs are usually performed using state duration sampling (SDS)
- Each component is assigned a mean failure rate and mean repair time
- Creating a state transition process of the up and down cycles of all system components.

## BUT

- The SE-reduced number of states ( $r$ ) **do not distinctly equate to an equivalent number of components.**
- For example,  $r = C_0^{97} + C_1^{97} = 98$  states for  $ED_1$  of the Single line model.
- This corresponds to a fictitious number of components between 6 ( $2^6 = 64$ ) and 7 ( $2^7 = 128$ ).
- **SO**, we use sequential MCS based on **state transition sampling (STS)**
- STS focuses on **state transitions** of the entire system, rather than on the individual component states
- The system will transit from one system state  $S_k$  into the next state  $S_{k+1}$  depending on the random state duration of the component that first departs from its present state (up or down) in system state  $S_k$

# Sequential MCS cont'd

The probability of the  $j^{th}$  component departing from present state at time  $t_0$  is:

$$P_j = \frac{\lambda_j}{\sum_{i=1}^m \lambda_i}$$

$i = 1, 2, \dots, m$ , where  $m$  is the number of components

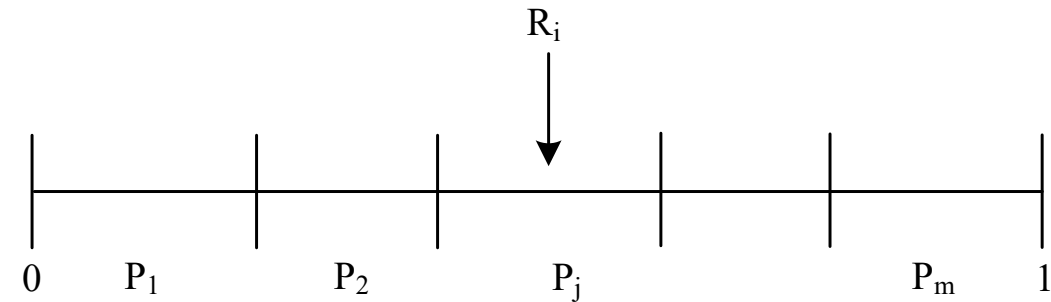
→ The state transition of any component leads to a state transition.

For  $m$  components, there can be  $m$  possible reached states.

→ The probability of the  $m$  states that could be reached can then be successively placed in the interval  $[0,1]$  (because  $\sum_j^m P_j = 1$ )

→ Assess the consequences of each system state, i.e., the impact on frequency and duration of customer interruptions.

→ Simulation is repeated until the required accuracy  $\varepsilon$



Generate a uniformly distributed random number  $R_i$  between 0 and 1.

$$\varepsilon = \sqrt{\text{var}(x)} / (\bar{x} \cdot \sqrt{N})$$

# Formulation of Reliability Equivalents of LV Networks



- Formulate **Unavailability ( $U$ )**, using **Energy Not Supplied (ENS)**.
- ENS combines both frequency- and duration-based indices.
- It is a **composite reliability performance indicator** that quantifies the combined effects of the numbers and durations of supply interruptions with the amount of interrupted demand.

$$\lambda_{eqv} = SAIFI \times LPS$$

$$U = \frac{\text{Annual ENS}}{\text{Connected demand} \times \text{hours in a year}}$$

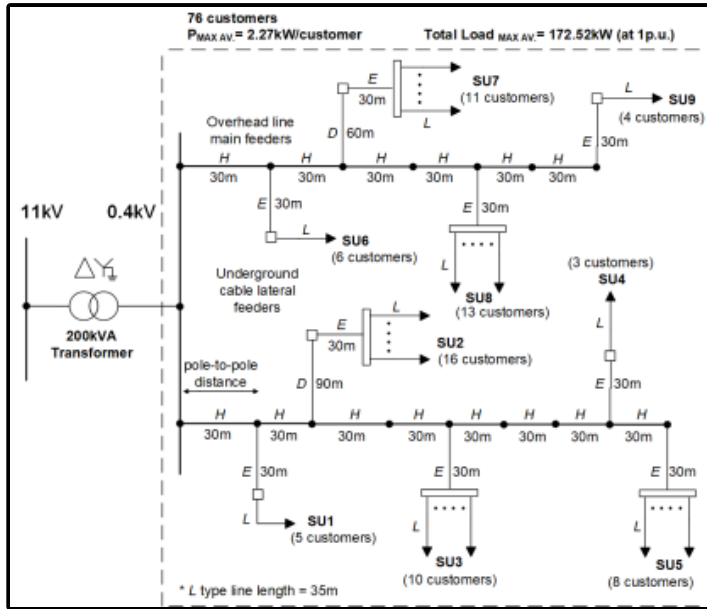
$$U = \frac{\lambda_{eqv}}{\lambda_{eqv} + \mu_{eqv}}$$

$$\mu_{eqv} = \frac{(1-U) \cdot \lambda_{eqv}}{U}$$

$$MTTR_{eqv} = \frac{1}{\mu_{eqv}}$$



# State Enumeration and Monte Carlo Simulation Techniques



- Detailed LV model
- Time-consuming MCS

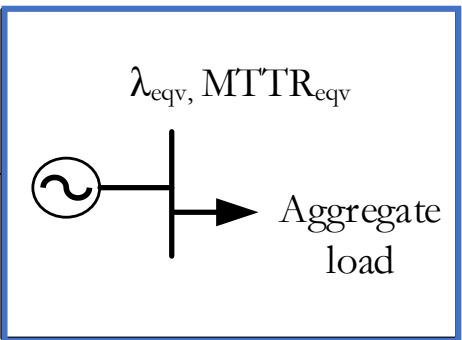
SE+MCS

**ED System States**

Reduced number of states  
 and accurate reliability indices

- Minimum error
- Faster MCS

Equivalent



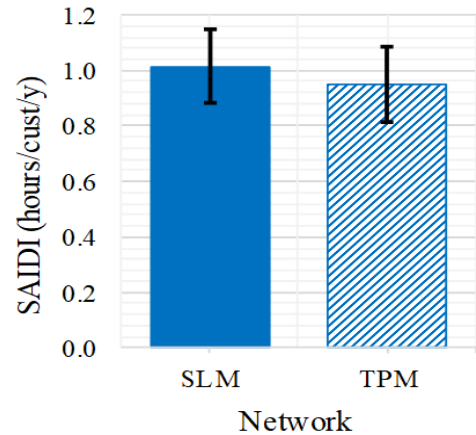
- Equivalent PC
- Same unavailability



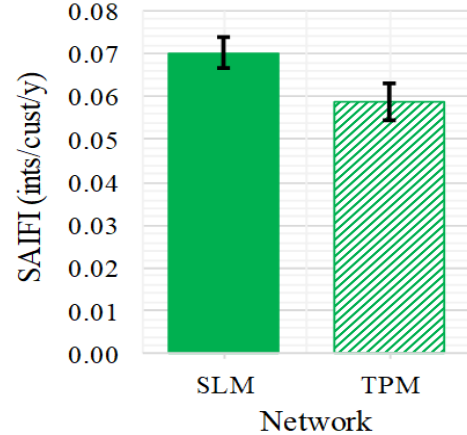
TPM Single Equivalent Component Parameters

Eqv-PC	$\lambda_{eqv}$ (failures/year)	$MTTR_{eqv}$ (hours)	Computational Time Saving (%)
TPM	1.555	0.588	99.98%

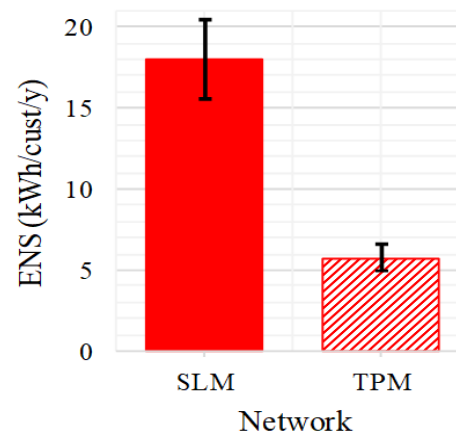
# Single-Line (1-ph) vs Three-Phase (3-ph) Models



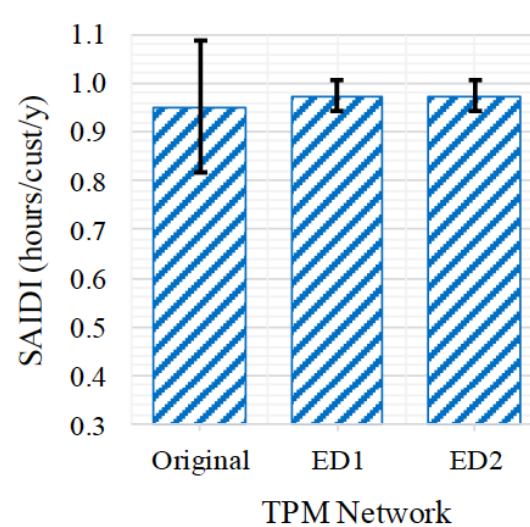
(a) SAIDI



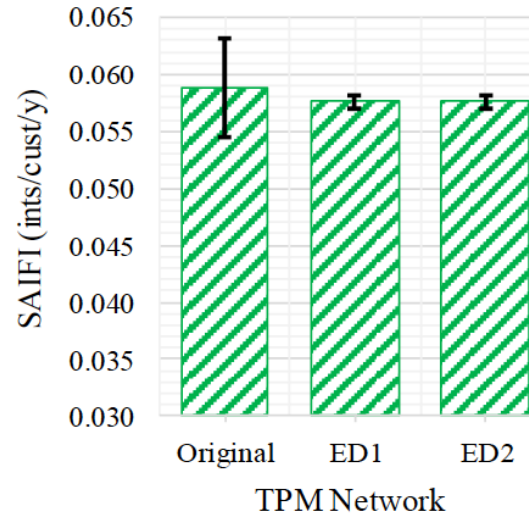
(b) SAIFI



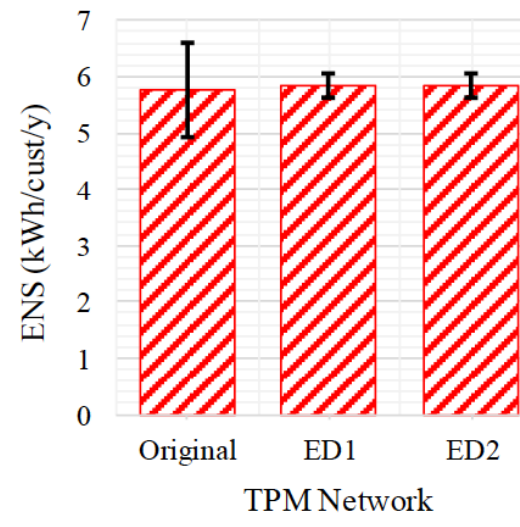
(c) ENS



(a) SAIDI



(b) SAIFI



(c) ENS

- TPM is **more accurate**;
- Detailed representation of the actual network;
- SLM **underestimates** reliability performance, mainly for ENS;
- TPM differentiates different fault types.

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- Low-order EDs are **sufficient** to assess reliability performance with **high accuracy**, while requiring significantly **shorter computational times**.

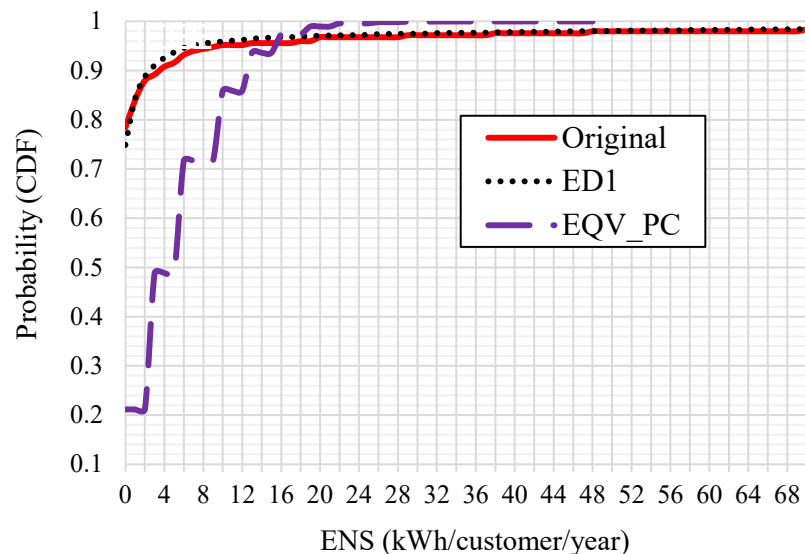
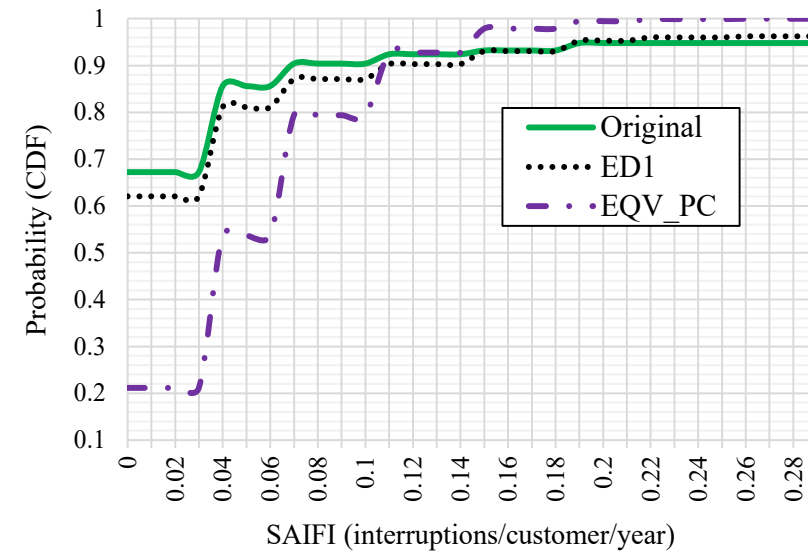
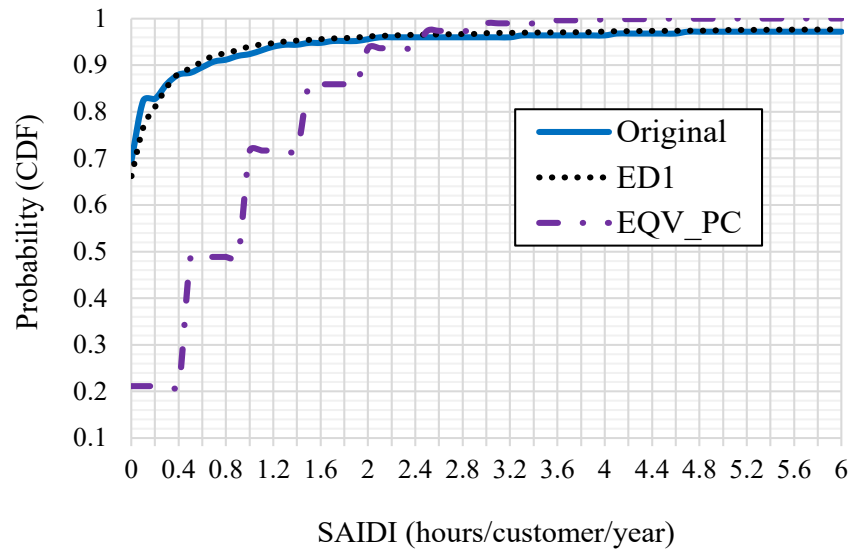
# Combining SE and SMCS for LV Reliability Equivalents

- $ED_1$  and  $ED_2$  produce **nearly identical** results - due to the low probability of double faults.
- $ED_2$  results in much longer simulation times than for  $ED_1$
- Computational time is reduced by **99.9% for  $ED_1$**  and by **91.1% for  $ED_2$**  models compared to original network.
- The time required to perform **equivalenting** is also higher in  $ED_2$  (33.8 hours) than  $ED_1$  (2.5 hours)
- $ED_1$  is **sufficiently representative** of the original network.

*Reliability Indices for the TPM Equivalent Component.*

Network	SAIDI		SAIFI		ENS	
	hrs/c/y	Error	ints/c/y	Error	kWh/c/y	Error
Orig.	0.9508	-	0.0588	-	5.7648	-
$ED_1$	0.9739	2.4%	0.0576	2.0%	5.8437	1.4%
Eqv-PC	0.9154	3.7%	0.0577	1.9%	5.8485	1.5%

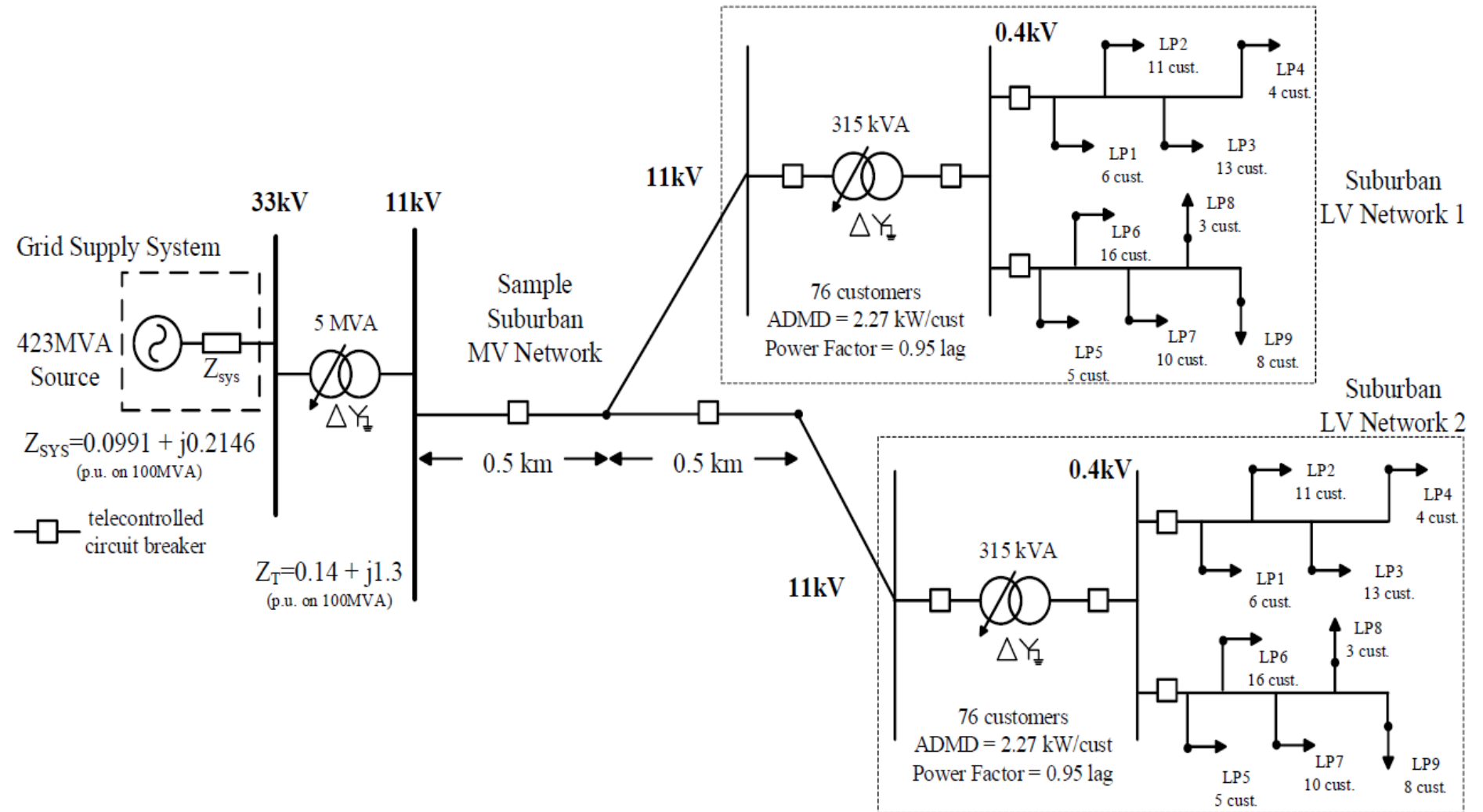
# Combining SE and SMCS for LV Reliability Equivalents



- Combined SE–SMCS ( $ED_1$ )  $\equiv$  *Original network*
- Eqv-PC aggregates composite reliability information
- Still adequately approximates the CDFs of the detailed original network with a higher number of components.
- Quantifying the risk of longer interruption times, frequency and ENS.

# Analysis at MV Level with LV Network Equivalents

## “Plug and Play” functionality



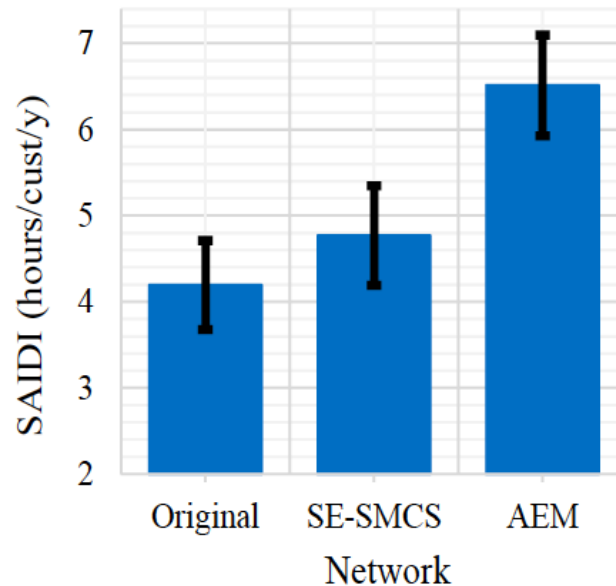
Original (SLM) suburban MV network model with 213 components

# Comparison of Results (at MV) of Two Single-Component (LV) Models

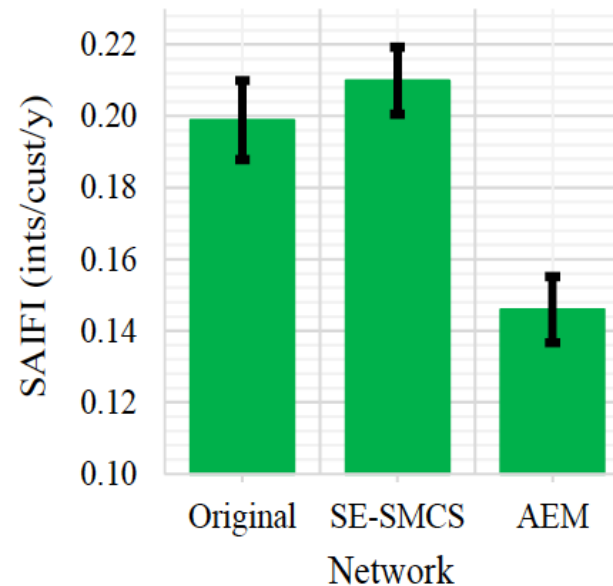
Eqv-PC	$\lambda_{eqv}$ (failures/year)	MTTR <sub>reqv</sub> (hours)
SE-SMCS	1.052	1.217
AEM	0.472	6.235

## SE-SMCS approach:

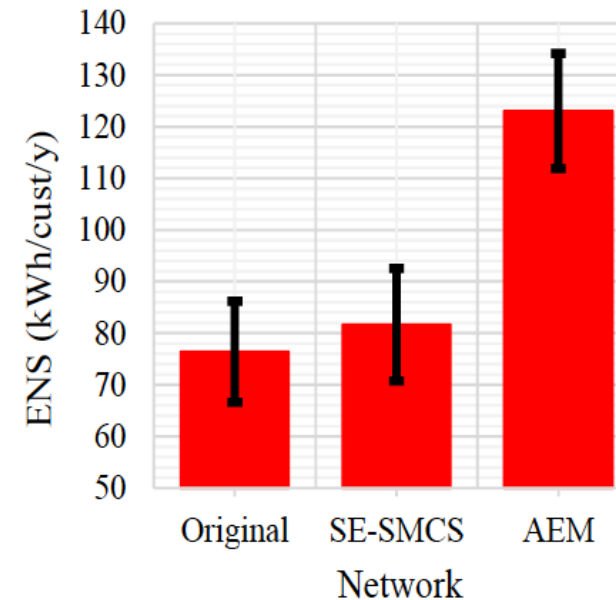
- Location of components;
- Impact of failures;
- Does not increase complexity;
- Reliability dependency between MV and LV networks.



(a) SAIDI



(b) SAIFI



(c) ENS

# Conclusions

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- A novel comparison of the (typically used) simplified SLMs of LV networks, in contrast with fully detailed TPMs, which **avoids systems' performance overestimation**.
- Development of a novel LV/MV network reliability assessment methodology that **combines SE and SMCS** to significantly reduce computational complexity while preserving accuracy.
- Development of a novel and simple **single-component network equivalent**, which offers the same **unavailability** (and therefore reliability performance) as the original LV/MV network.
- Accuracy, computational efficiency and scalability of the proposed LV equivalents is tested and validated in more complex and larger MV networks, for **replicability** of the proposed methodology.

# Thanks for listening. Any Questions?

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